Labor Market Regulation and Retirement Age

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Abstract

Present paper analyzes the determinants of political consensus on relevant aspects of the Social Security System and focus on the choices over employment protection and retirement age; a theoretical model is build where an equilibrium setup results from a political process involving three social groups: young, low and high productivity old. Hypothesis and results of the model are tested using macro data. The aim of the analysis is to provide some insight on the reasons why some institutional setups are supported by voters and implemented while other don’t

JEL Classification: D72, H55, J63.
1 Introduction

Pension system and labor market reforms are widely debated issues in all industrialized countries and specially in Europe; the demographic trend and the recent slowdown of economic growth in the EU indeed, urge politicians to revise the setup of Social Security Systems.

A prominent role in this debate is played by employment protection regulation and by mandatory retirement age. A reform of the labor market involving a reduction in employment protection is considered an important tool to promote economic growth; an increase in retirement age is a key element to guarantee the viability of the pension system in the long run.

The present work considers Social Security from a wide point of view that includes in the picture mutual connections between labor market and the pension system; this approach provides useful insights on the reasons why some institutional setups are supported by voters and implemented while other don’t.

The aim of the paper is to describe the determinants of political consensus on relevant aspects of the Social Security System and to provide some predictions over the feasibility of different reforms. The analysis focus on the interaction between employment protection and retirement age; an equilibrium setup for these elements results from a political process involving three social groups: young, low productivity and high productivity old.

Two factors are crucial in this setting: total turnover on the labor market and unemployment risk; in particular, the first element affects income distribution across time while the second one changes distribution across states of the world.

The degree of employment protection defines the number of exits from the labor market due to workers selection at the firm level; in other words, it sets total turnover between insiders and outsiders. The young generation plays the role of the outsiders, therefore a change in employment protection produces also a different allocation between current and old age income.

Low productivity old insiders consider employment protection as an insurance device against the risk of being unemployed; the selection process indeed, affects only these agents. High productivity old that do not risk to be fired, are mainly concerned about their chances to get back to work if they lose the job for reasons other than selection; a lower protection increases their probability to be hired again and reduces unemployment risk.

Mandatory retirement sets the moment in time when a complete generational turnover happens due to permanent non-selective exits of old insiders; this again affects young workers’ intertemporal income allocation.

The old generation considers retirement as an insurance device to reduce the maximum length of unemployment periods; when workers retire they substitute wage with pension and a trade-off arises between insurance and total expected income, if the replacement rate is less than one.

Employment protection and retirement age meet agents’ needs differently and produce also different costs. Old workers’ retirement requires to be financed via social security contributions in a pay-as-you-go system; a reduction in employment protection instead, does not involves monetary expenses.
On the other hand, while retirement realizes a complete turnover between old and young workers, the efficiency of the selection process depends on several factors. The main issue is the productivity gap existing between insiders being fired and outsiders being hired; this variable results both from average human capital in the population and from skill mismatches between labor demand and supply.

The young generation is pivotal in the political process; when the productivity gap widens, employment protection increases while retirement age decreases. If the gap is high the selection process has little impact on total turnover; many low productivity workers are substituted by few high productivity agents; early retirement is more effective than low employment protection in redistributing young workers’ income across time.

Outsiders’ preferences in this case, are similar to those of low productivity old that mostly favor early retirement to be insured against unemployment; the consensus of both groups converges toward a setup including high employment protection and early retirement.

When the productivity gap is low, the selection process is particularly effective in increasing labor market turnover; intertemporal income redistribution then, is realized reducing employment protection.

As a consequence the preferences of the young get close to those of high productivity old workers; enhancing selection opportunities indeed, increases the level of insurance against the unemployment risk of this last type of agents. In order to minimize pension system costs moreover, both groups agree over a setup for the Social Security System where no employment protection is implemented together with late retirement.

The approach of present analysis is twofold: first a theoretical model is built to analyze the determinants of social consensus over different setups of the Social Security System; the second step is to test hypothesis and results of the model through the use of macro data.

2 Related Literature

A number of papers considers the relationship between the pension system and other aspects of Social Security in a political economy framework; the extensive work of Mulligan, Sala-i-Martin (1999a, 1999b) provides an exhaustive compendium of the main issues at a stake.

Conde-Ruiz and Galasso (2003) analyze public pensions and retirement age; they find that, in equilibrium, the political support for a large Social Security System relies on elderly with incomplete working history and low-ability young that expect to retire early.

In Galasso and Conde-Ruiz (2000) pension and redistribution systems are considered jointly; in equilibrium a welfare state characterized by generous public pensions and a high level of income redistribution, is backed by elderly and low-income young. Similar results are found also in Lambertini and Azariadis (1998) where unskilled workers and retirees form the winning coalition.
Another strain of literature considers mutual interactions between labor market and the Social Security System.

Boeri, Conde Ruiz and Galasso (2003) focus on labor market risks considering in particular, the trade-off between employment protection legislation and unemployment benefits; they show that two main outcomes emerge from the political process. The first one is characterized by low unemployment benefits and high employment protection and arises if low-skill insiders are a majority; the second one includes high unemployment benefits and low employment protection.

Recently, Brugiavini, Conde Ruiz and Galasso (2003) did consider the effects of income transfers from parents to kids; the tax rate financing the Social Security System depends, in this setting, on the number of unemployed workers in the economy. When the unemployment rate is high, a large number of old parents are recipient of the welfare state; kids are less likely to find a job and rely heavily on transfers from them. A coalition between old recipient of the welfare state and non-emancipated kids supports a large Social Security System; the opposite happens if the unemployment rate is low.

The present work contributes to this debate considering employment protection legislation and retirement age as the outcomes of a unique political process; other papers analyzed such aspects of the Social Security System, but not jointly.

3 The Model

3.1 The Economy

Consider an economy where each agent lives for two periods and is young in the first and old in the second. There is no population growth and each generation counts the same number of individuals; the size of one cohort is normalized to one.

Agents differ for their labor productivity: in each generation, half are high productivity workers and half are low productivity. At time $t$ old individuals’ type is common knowledge; the productivity of the young is observed only when they start looking for a job.

Workers can be classified in three different types according to their age and productivity: high productivity old, low productivity old and young workers.

Utility is a function $\nu(c_t, c_{t+1})$ of present, $c_t$, and future consumption, $c_{t+1}$; in particular, it is the case that:

$$\nu(c_t, c_{t+1}) = u(c_t) + \frac{1}{2} \cdot u(c_{t+1})$$

where $\frac{1}{2}$ is the intertemporal discount factor.

There are no saving means and output is not storable; all that is produced within a period is also consumed.

1 Equivalent results are obtained by setting the intertemporal discount factor $\beta$ at a level such that:
Utility is influenced exclusively by own consumption; both intra-generational (between high and low productivity old) and inter-generational (between old and young workers) altruism are absent.

One representative firm produces a unique good that is also the numeraire; the final good market is competitive and any quantity can be sold at the equilibrium price.

The production function uses labor as the only input and is defined as:

\[ Y_t = (a \cdot L_{t}^{HP} + L_{t}^{LP})^\alpha \]

where \(L_{t}^{HP}\) and \(L_{t}^{LP}\) are respectively the number of high and of low productivity agents employed at time \(t\); the parameters \(a \geq 1\) and \(\alpha \in [0, 1]\) describe the productivity gap between the two types of workers and the technology in use.

There are two minimum wage levels: one for low productivity workers

\[ W_{t}^{LP} = \frac{\alpha}{\frac{1}{a+1}} \]

and one for high productivity agents, \(W_{t}^{HP} = a \cdot W_{t}^{LP}\).

Except for these constraints, labor market is competitive; a unique market exists where units of efficient labor, \(x_t\), are exchanged and where each of them cannot be paid less than \(W_{t}^{LP}\). The production function then, can be rewritten as:

\[ Y_t (x_t) = (x_t)^\alpha \]

The representative firm chooses the quantity of labor that maximizes its profits \(\Pi_t\), defined as:

\[ \Pi_t = Y_t - W_{t}^{LP} \cdot x_t \]

Despite the fact that one unit of labor has the same price, no matter if the supplier is a high or a low productivity worker, it is assumed that the representative firm hires high type individuals first; this is the case, for instance, because they improve the technology in use through the introduction of more effective routines\(^2\).

\[ \beta > \log \left[ \frac{\left( \frac{1-p}{1-p}\right)^2}{\frac{1}{3} + \frac{p}{3} + \frac{p^2}{3} + \frac{p^3}{3}} \right] \]

where

\[ \log \left[ \frac{\left( \frac{1-p}{1-p}\right)^2}{\frac{1}{3} + \frac{p}{3} + \frac{p^2}{3} + \frac{p^3}{3}} \right] \leq \beta \]

\(^2\) Representative firm preference for high productivity workers is not explicitly modeled in this setting; it can be shown though, that this setting is an approximation for the case where these agents provide a reduction in production costs that approaches zero. A formal proof is found in the appendix.
All agents have no disutility from working and labor supply is perfectly rigid. Old individuals work for a fraction $\frac{1+\theta}{2}$ of the period and retire in the last fraction $\frac{1-\theta}{2}$ of it; the young generation remains on the labor market for the whole period. There are no unemployment benefits and workers get nothing if they do not have a job.

The Social Security System awards to each retired old a fraction $p \in (0, 1]$ of his last wage. Pensions are financed by taxation on labor income; the tax rate, $\tau_t$, is such that a balanced budget condition holds.

### 3.2 Timing and Structure of the Game

The unitary period is divided into two sub-periods of the same size; there are four stages in the game, two for each sub-period.

#### 3.2.1 First Sub-period

At time 0 the first stage begins and the political process takes place; the parameters $\eta$ and $\theta$ are chosen, once and for all, after that. The first variable sets the fraction of low productivity workers that the representative firm is allowed to fire each period; the second one defines the fraction of the period that an old agent actually works before retirement.

After the political process, the second stage starts; the young start looking for jobs and reveal their type (suppose for instance, they completed a schooling path that is a perfect signal of productivity) while a stochastic displacement process hits the insiders. This causes half of the employed old workers to leave the job for reasons that are not covered by employment protection such as: bankruptcy of the firm, illness, need to move to other places, serious mistakes or fraud, voluntary unemployment or maternity etc.; being stochastic, the displacement process affects in the same measure high and low productivity agents.

The representative firm then, chooses the input quantity, $x_1^0$, to set the outflow in labor due to displacement. High productivity individuals are hired first; the number of incoming workers is such that, given the composition of the unemployment pool between high and low types, the quantity of labor amounts to $x_1^0$.

#### 3.2.2 Second Sub-period

In the second sub-period, at date $\frac{1}{2}$, the third stage begins; a selection process takes place where, according to the outcome of the political process, a fraction $\eta$ of low productivity workers is fired.

The representative firm then, chooses the quantity, $x_2^0$, to get from the market to offset the reduction in labor due to the selection process; again high productivity individuals are chosen first and the number of workers hired depends on the composition of the unemployment pool.
After a fraction $\theta$ of this sub-period the old retire; at date $\frac{1+\theta}{2}$ the stock of labor employed in the production decreases again. The representative firm chooses the input quantity, $x^3_0$, to get from the market; the same procedure described before defines the hiring process.

The graph that follows displays the timing of the model:

Graph 1: Timing of the Model

<table>
<thead>
<tr>
<th>Voting / Displacement process</th>
<th>Selection process</th>
<th>Mandatory retirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(1+\theta)$</td>
</tr>
</tbody>
</table>

Consider now the implications of this setting on the pension system.

Notice that, since taxes are levied on labor income, whenever a young agent gets a job he contributes to the Social Security System; therefore if $\theta < 1$ retirement involves a net income transfer from the young to the old generation.

When $\eta \neq 0$ and $\theta < 1$, high productivity old agents are more likely to be employed than low productivity ones and also their expected contributions are proportionally higher; the system then, defines also a transfer from the first to the second type of agent.

### 3.3 The Political Process

The degree of protection of insider workers, $\eta$, and the mandatory retirement age, $\theta$ result from the political process; these two variables are discrete and it is the case that $\eta \in \{0; 1\}$ and $\theta \in \{0; \frac{1}{2}; 1\}$. The political process is an open agenda majority voting where each bidimensional platform $(\eta, \theta)$ is compared with one alternative at a time.

A problem in this setting arises: since the young generation represents half of the electorate, ties are possible if the whole old generation casts the same vote. When this happens it is assumed that the choice of the old prevails; this
is coherent with the findings of a number of papers where the pressure exerted by the elderly drives the outcomes of the political process.3

The platform in the set \( \{(0,0);(0,\frac{1}{2});(0,1);(1,0);(1,\frac{1}{2});(1,1)\} \) that gets a majority of votes (or is chosen by the whole old generation) against any other alternative is the Condorcet winner of the game and is implemented.

### 3.4 Agents’ Preferences

Given previous description of the economy, consider now the maximization problem of each type of worker.

#### 3.4.1 High Productivity Old Workers

The maximization problem of a high productivity old worker (HP) is:

\[
\begin{align*}
\text{MAX}_{\eta \in \{0;1\}, \theta \in \{0,\frac{1}{2}, 1\}} & \ U^\text{HP}_0 (\eta, \theta) \\
\text{where} & \\
U^\text{HP}_0 (\eta, \theta) & = \frac{1}{2} \cdot u \left[ \frac{1 + \theta}{2} \cdot a \cdot W^{LP} (1 - \tau) + \frac{1 - \theta}{2} p \cdot a \cdot W^{LP} \right] + \\
& + \frac{1}{2} \cdot \frac{\eta}{2a - 1} \cdot u \left[ \frac{\theta}{2} \cdot a \cdot W^{LP} (1 - \tau) + \frac{1 - \theta}{2} p \cdot a \cdot W^{LP} \right] + \\
& + \frac{1}{2} \left( 1 - \frac{\eta}{2a - 1} \right) u \left[ \frac{1 - \theta}{2} p \cdot a \cdot W^{LP} \right]
\end{align*}
\]

Expected utility is the summation of three elements; each refers to a specific working history of the agent.

The first one describes the event in which an high productivity worker is not displaced in the first sub-period and works until retires.

The second element refers to the situation where the agent is displaced in the first sub-period, but hired after the selection process; he works for a fraction \( \theta \) of the second sub-period and then retires.

The last one corresponds to the case where the worker is displaced in the first sub-period and never hired again; he gets only the pension for the fraction \( (1 - \theta) \) of the second sub-period.

#### 3.4.2 Low Productivity Old Workers

The maximization problem of a low productive old worker (LP) is:

\[3\text{See Galasso and Profeta (2002) for a survey over this issue.}\]
\[ MAX_{\eta \in \{0,1\}, \theta \in \{0,\frac{1}{2},1\}} U_{0}^{LP} (\eta, \theta) \]

with

\[ U_{0}^{LP} (\eta, \theta) = \frac{1}{2} (1 - \eta) u \left[ \frac{1 + \theta}{2} \cdot W_{LP} (1 - \tau) + \frac{1 - \theta}{2} p \cdot W_{LP} \right] + \]
\[ + \frac{\eta}{2} u \left[ \frac{1}{2} \cdot W_{LP} (1 - \tau) + \frac{1 - \theta}{2} p \cdot W_{LP} \right] + \]
\[ + \frac{1}{2} u \left[ \left( 1 - \frac{1 - \theta}{2} p \right) \cdot W_{LP} \right] \]

Expected utility is the result of agents’ working history; a low productivity worker may face three different situations corresponding to the elements of the summation in the above equation.

The first situation corresponds to the case where he is not displaced in the first sub-period and further is not fired in the next one; the agent works until retires.

In the second case the worker is not displaced in the first sub-period, but is fired in the second one; he works only in the first sub-period and then is unemployed until he retires.

In the third situation the agent, displaced in the first sub-period, is never hired again; he is unemployed until retirement.

### 3.4.3 Young Workers

The young maximize expected utility across two periods and solve the following problem:

\[ MAX_{\eta \in \{0,1\}, \theta \in \{0,\frac{1}{2},1\}} U_{0}^{Y} (\eta, \theta) = U_{0}^{Y} (\eta, \theta) + \frac{1}{4} \cdot U_{1}^{HP} (\eta, \theta) + \frac{1}{4} \cdot U_{1}^{LP} (\eta, \theta) \]

where

\[ U_{0}^{Y} (\eta, \theta) = \frac{1}{2} u \left[ \frac{(1 - \theta)}{2} \cdot W_{LP} (1 - \tau) \right] + \frac{a + 1}{4a} u \left[ a \cdot W_{LP} (1 - \tau) \right] + \]
\[ + \frac{1}{2} \left( 1 - \frac{a + 1}{2a} \right) \left( 1 - \frac{\eta}{2a - 1} \right) u \left[ \frac{(1 - \theta)}{2} \cdot a \cdot W_{LP} (1 - \tau) \right] + \]
\[ + \frac{1}{2} \left( 1 - \frac{a + 1}{2a} \right) \left( 1 - \frac{\eta}{2a - 1} \right) u \left[ \left( 1 - \frac{1 - \theta}{2} p \right) \cdot a \cdot W_{LP} (1 - \tau) \right] \]

Notice that since saving means are absent and output is not storable, no transfers across periods are possible; each agent relies on current income to finance his consumption.

Expected utility in the first period, \( U_{0}^{Y} (\eta, \theta) \), results from the summation of four elements.
The first of them describes the situations where the worker happens to be a low productivity type: he is hired only when all old agents retire and works for a fraction \( \frac{1-\theta}{2} \) of the whole period.

The remaining elements refer to the case where the agent is a high productivity worker; in particular, the second of them describes the situation where he is hired in the first sub-period, after displacement, and works during the whole period.

The third one corresponds to the case where the worker is hired at the beginning of the second sub-period through the selection process.

The last element is equivalent to the first one; the worker is hired only after retirement of old agents.

Young agents’ expected income over the two periods is constant and amounts to \( \frac{w_{LP}}{2} (a + 1) \); the choice of a platform \((\eta, \theta)\) defines the optimal resource allocation across time and states of the world.

Notice that a reduction in employment protection increases the probability that these agents get a job after the selection process if they are high productivity types; this circumstance produces an intertemporal redistribution of income, since young workers are more easily hired in the first period.

On the other hand an increase in \( \eta \) involves also a different income allocation across states of the world; resources are diverted from the situations where the agent is low type and transferred to situations where he is instead, high type. A low degree of employment protection implies that the selection process is more effective in expelling less productive workers from the employment pool; this produces more opportunities for the high types to get back into employment when they are displaced or looking for their first job.

A higher retirement age causes an increase in future income and a reduction in current earnings; delaying the complete generational turnover induced by retirement indeed, shrinks the size of the period where all young agents are employed.

Cutting pension income through a reduction in the duration of retirement, produces also a decrease in old age income for less productive agents; this is the case because these workers shoulder on average, longer periods of unemployment.

4 Equilibrium Analysis

The game is sequential and requires backward induction to be solved; the equilibrium analysis starts then, from the maximization problem of the representative firm.
4.1 The Maximization Problem of the Representative Firm and the Dynamic of the Labor Market

Previous description of labor market allows to define the equilibrium strategy of the representative firm in a very simple way\(^4\).

Notice that marginal productivity of labor equals its minimum price per efficient unit when \(\frac{1}{2}(a+1)\) is the quantity employed in the production process; it is indeed:

\[
\frac{\partial Y_t(x_t)}{x_t} \bigg|_{x_t=\frac{1}{2}(a+1)} = \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} = W_{LP}
\]

Given population composition between high and low productivity agents, total labor input provided by one generation is \(\frac{1}{2}(a+1)\) and labor supply exceeds demand if two generations are on the market; when old workers retire then, the representative firm hires all the young and the labor market is in equilibrium.

This means that before the political process takes place old agents are the insiders of the labor market while the young are all unemployed; the minimum wage is set at the highest level that prevents the outsiders to get a job.

The wage rate moreover, is fixed every sub-period at \(W_{LP}\) for low productivity workers and \(W_{HP}\) for high productivity workers. Also labor demand does not change across time and the representative firm holds constant the input level at \(\frac{1}{2}(a+1)\); when labor is reduced by displacement or by the selection process, outflowing input is exactly replaced.

Consider now the dynamic of the labor market implied by the equilibrium strategy of the representative firm. If the level of input never changes, both employment and unemployment rates do; indeed, the number of agents working in the firm depends on inflows and outflows of high and low productivity workers.

The displacement process causes half of the insiders to lose their job at the beginning of each period; the employment pool includes the whole old generation and the expected reduction in input amounts to \(\frac{1}{4}(a+1)\).

Since is \(\frac{1}{4}(a+1) \leq \frac{1}{2}a\), high productivity young supply enough labor to meet the decrease in input; the number of them that is hired is \(\frac{1}{4}(\frac{2a+1}{a})\). This amounts to a probability \(\frac{1}{4}(\frac{2a+1}{a})\) to get a job after displacement.

Unemployment increases from 1 (the whole young generation) to \(\frac{5a-1}{4a} \geq 1\). Unemployment composition changes; high and low productivity workers do not represent 50% of the total each but amount respectively to \(\frac{2a-1-a}{4a} \) and \(\frac{3a-a}{4a}\).

At half of the period the number of low productivity workers employed in the representative firm is \(\frac{1}{8}\); the selection process then, causes a decrease in labor input amounting to \(\frac{1}{2}\).

Being \(\frac{2a-1-a}{4a} \geq \frac{1}{4}\), high productivity unemployed supply enough input to meet labor demand; the number of them that gets a job is \(\frac{a}{2a}\) and the probability to be hired that these workers, either young or old, face amounts to \(\frac{a}{2a}\). Unemployment increases to a total of \(\frac{5a-1}{4a} + \frac{a(a-1)}{4a}\) where \(\frac{2a-1-a}{4a}\) are high productivity and \(\frac{3a-a}{4a}\) are low productivity agents.

\(^4\) A formal derivation of this result is found in Appendix 1.
After retirement, all the young get a job and unemployment is zero.

Given the equilibrium dynamic of labor market, the balanced budget condition for the pension system requires:

\[ \tau \cdot \alpha \left( \frac{1}{2} (a + 1) \right)^\alpha = p \cdot \frac{1 - \theta}{2} \cdot \alpha \left( \frac{1}{2} (a + 1) \right)^\alpha \]

and the tax rate is set at:

\[ \tau = p \cdot \frac{1 - \theta}{2} \]

### 4.2 The Political Process

Consider now the political process; a Condorcet winner for the voting stage is an equilibrium for this stage of the game.

Notice that since the electorate is a continuum, each voter has zero mass and cannot affect the result of the election; strategic voting is excluded and it is possible to assume that each agent expresses his preferences sincerely.

When workers turn to vote they choose the platform \((\theta, \eta)\) that maximizes expected utility; in this setting a strategy for agent \(i\), \(\Sigma_i\), is a function of his type, \(\xi\), of the replacement rate, of the technology parameter and of the productivity gap between high and low productivity workers.

\[ \Sigma_i (\xi, p, \alpha, a) : \{HP; LP; Y\} \times [0, 1] \times [0, 1] \times [0, +\infty) \rightarrow \{0; \frac{1}{2}; 1\} \times \{0; 1\} \]

The definition of the equilibrium requires to look at the preferences of each type of agent. The analysis that follows considers the case where agents are risk averse; both employment protection and retirement age in fact, are used to insure against unemployment risk.

In this framework, agents’ maximization problems are not easily solved analytically unless a specific assumption on agents’ preferences is introduced; in order to get a closed form characterization of the equilibrium, it is required that agents’ utility function is logarithmic in income.

#### 4.2.1 High productivity old workers

The maximization problem of these agents is:

\[
\text{MAX}_{\eta \in \{0; 1\}, \theta \in \{0; \frac{1}{2}; 1\}} \log \left( \frac{a \cdot W^{LP}}{2} \right) + \frac{1}{2} \log \left[ 1 + \theta + \frac{p}{2} (1 - \theta)^2 \right] + \\
+ \frac{1}{2} \left( \frac{\eta}{2a - 1} \right) \log \left[ \theta + \frac{p}{2} (1 - \theta) (2 - \theta) \right] + \frac{1}{2} \left( 1 - \frac{\eta}{2a - 1} \right) \log \left[ p (1 - \theta) \right]
\]

**Proposition 1**

High productivity old workers’ preferred platforms are:

- \((1, \frac{1}{2})\) if \(a < \frac{1}{2}\)

\[ 1 + \frac{\log \left( \frac{\theta + \frac{p}{2}}{2a + \frac{p}{2}} \right)}{\log \left( \frac{\theta + \frac{p}{2}}{2a + \frac{p}{2}} \right)} \]
Proof. The first derivative of high productivity workers’ expected utility with respect to η is:

\[
\frac{1}{2} \left( \frac{1}{2a - 1} \right) \log \left[ 1 - \frac{\theta}{2} + \frac{\theta}{p(1 - \theta)} \right] \geq 0
\]

this means that if θ > 0, \( \frac{\delta U_{HP}(\eta, \theta)}{\delta \eta} > 0 \) holds, while it is \( \frac{\delta U_{HP}(\eta, \theta)}{\delta \eta} = 0 \), if \( \theta = 0 \); as a consequence \( \eta = 1 \) is preferred to \( \eta = 0 \) anytime \( \theta > 0 \) and choosing \( (0, 1) \) or \( (0, \frac{1}{2}) \) is not optimal. The platforms \( (1, 0) \) and \( (0, 0) \) moreover, provide the same expected utility.

If \( \theta = 1 \), \( U_{0}^{HP}(\eta, 1) = -\infty \) holds and the platform \( (1, 1) \) can be discarded.

Only \( (1, 0) \), \( (0, 0) \) and \( (1, \frac{1}{2}) \) then, are possible solutions for the maximization problem; in particular, the last alternative is preferred to the others if it is \( U_{0}^{HP}(1, \frac{1}{2}) - U_{0}^{HP}(1, 0) \geq 0 \) i.e. if:

\[
\frac{1}{2} \left( \frac{1}{2a - 1} \right) \cdot \log \left( \frac{1}{p} + \frac{3}{4} \right) - \frac{1}{2} \cdot \log \left( \frac{4 + 2p}{3 + \frac{3}{4}} \right) \geq 0
\]

Solving previous inequality for \( a \) gives:

\[
a < \frac{1}{2} \left[ 1 + \frac{\log \left( \frac{1}{p} + \frac{3}{4} \right)}{\log \left( \frac{4 + 2p}{3 + \frac{3}{4}} \right)} \right]
\]

When the above condition holds then, the solution for high productivity old workers’ maximization problem is \( (1, \frac{1}{2}) \); if instead, the productivity gap exceeds this threshold, \( (1, 0) \) and \( (0, 0) \) are chosen.

Consider now the main determinants of previous results.

No employment protection always provides insurance against the unemployment risk due to displacement.

If the productivity gap is large, the probability to get a job from the representative firm through the selection process is low; decreasing retirement age then, turns out to be beneficial especially when the replacement rate is high. A symmetric arguments apply to the case where the productivity gap is small.

4.2.2 Low productivity old workers

The maximization problem faced by a low productivity old worker with logarithmic utility is:

\[
\text{MAX}_{\eta \in \{0, 1\}, \theta \in \{0, \frac{1}{2}, 1\}} \log \left( \frac{W_{LP}}{2} \right) + \frac{1 - \eta}{2} \log \left[ 1 + \theta + \frac{p}{2} (1 - \theta)^2 \right] + \\
+ \frac{\eta}{2} \log \left[ 1 + \frac{p}{2} (1 - \theta) \right] + \frac{1}{2} \log \left[ p(1 - \theta) \right]
\]
Proposition 2  Low productivity old workers preferred platforms are \((1, 0)\) and \((0, 0)\).

Proof.  Look first at the first derivative of expected utility with respect to \(\eta\):

\[
-\frac{1}{2} \log \left[ 1 + \theta \cdot \frac{1 - \frac{\eta}{2} (1 - \theta)}{1 + \frac{\eta}{2} (1 - \theta)} \right] \leq 0
\]

Marginal utility of \(\eta\) is always negative if \(\theta > 0\); the platforms \((1, 1)\) and \((1, \frac{1}{2})\) then, are not solutions for the maximization problem of these agents. Also in this case moreover, when \(\theta = 0\), low productivity old workers are indifferent to the degree of employment protection; the alternatives \((0, 0)\) and \((1, 0)\) provide the same expected utility.

Notice that for \(\theta = 1\) expected utility is \(U^{LP}_0(\eta, 1) = -\infty\) and the alternative \((0, 1)\) is not an optimal choice. The platforms \((0, 0)\) (or equivalently \((1, 0)\)) and \((0, \frac{1}{2})\) are possible solutions for the maximization problem.

Consider the first derivative of expected utility with respect to \(\theta\):

\[
\frac{\delta U^{LP}_0(\eta, \theta)}{\delta \theta} = \left( \frac{1 - \eta}{2} \right) \frac{1 - p (1 - \theta)}{1 + \theta + \frac{\eta}{2} (1 - \theta)^2} - \left( \frac{\eta}{2} \right) \frac{p}{1 + \frac{\eta}{2} (1 - \theta)^2} - \frac{1}{2} \cdot \frac{1}{1 - \theta}
\]

and notice that for \(p \leq 1\):

\[
\frac{\delta U^{LP}_0(\eta, \theta)}{\delta \theta} \mid_{\eta = 0} = \frac{1}{2} \cdot \frac{1 - p (1 - \theta)}{1 + \theta + \frac{\eta}{2} (1 - \theta)^2} - \frac{1}{2} \cdot \frac{1}{1 - \theta} \leq 0
\]

holds; this means that \(\theta = 0\) is an optimal choice whenever \(\eta = 0\). Low productivity old workers then, prefer the platforms \((0, 0)\) and \((1, 0)\) to \((0, \frac{1}{2})\) and to all other alternatives. 

Consider now the determinants for the preferences of this kind of agents.

Complete employment protection represents the most effective insurance device against unemployment caused by the selection process; as a consequence low productivity old prefer this solution when \(\theta \neq 0\).

The pension system instead, reduces the risk being unemployed until retirement if displaced; since no income is earned when this happens, complete insurance is chosen and \(\theta\) is set at zero.
4.2.3 Young workers

The maximization problem for a young worker is the following:

\[
\max_{\eta \in \{0,1\}, \theta \in \{0,1\}} \log \left( \frac{W_{LP}}{2} \right) + \frac{1}{2} \log a + \log \left[ 1 - \frac{p}{2} (1 - \theta) \right] + \\
+ \frac{1}{2} \left[ 1 + \left( \frac{a-1}{2a} \right) \left( 1 - \frac{\eta}{2a-1} \right) \right] \log (1 - \theta) + \frac{1}{4} \cdot \frac{a+1}{a} \log 2 + \\
+ \frac{1}{4} \left\{ \log \left( \frac{a \cdot W_{LP}}{2} \right) + \frac{1}{2} \log \left[ 1 + \theta + \frac{p}{2} (1 - \theta)^2 \right] + \\
+ \frac{\eta}{2} \left( \frac{1}{2a-1} \right) \log \left( 1 - \frac{p}{2} (1 - \theta) (2 - \theta) \right) + \frac{1}{2} \left( 1 - \frac{\eta}{2a-1} \right) \log [p (1 - \theta)] \right\} + \\
+ \frac{1}{4} \left\{ \log \left( \frac{W_{LP}}{2} \right) + \frac{1}{2} \log \left[ 1 + \theta + \frac{p}{2} (1 - \theta)^2 \right] + \\
+ \frac{\eta}{2} \log \left( 1 + \frac{p}{2} (1 - \theta) \right) + \frac{1}{2} \log [p (1 - \theta)] \right\}
\]

Notice that, as it was the case for old workers, also young agents are indifferent to the degree of employment protection when \( \theta = 0 \); indeed it is:

\[
U_Y^0 (0, 0) = U_Y^0 (1, 0) = \\
\frac{3}{2} \log \left( \frac{W_{LP}}{2} \right) + \frac{3}{4} \log a + \left( \frac{a+1}{4a} \right) \log 2 + \frac{1}{4} \left[ \log \left( 1 + \frac{p}{2} \right) + \log p \right]
\]

A second relevant observation is reported in the following proposition.

**Proposition 3** For \( 0 < p \leq 1 \) there always exists a level of productivity gap, \( a^* \), such that for every \( a > a^* \), \( U_Y^0 (0, 0) - U_Y^0 (1, \frac{1}{2}) \geq 0 \) holds.

**Proof.** In order to have \( U_Y^0 (0, 0) - U_Y^0 (1, \frac{1}{2}) \geq 0 \), the following inequality must hold:

\[
\frac{1}{2} \left[ \frac{19}{4} + \frac{(a-1)}{a(2a-1)} \right] \log 2 - \log \left( \frac{4-p}{2-p} \right) + \\
\frac{1}{8} \left[ \log \left( \frac{12+p}{2+2p} \right) + \frac{1}{2a-1} \log \left( \frac{4+3p}{p} \right) + \log \left( \frac{4+p}{2+p} \right) \right] \\
\geq 0
\]

For \( a \geq 1 \) it is:

\[
\frac{\delta [U_Y^0 (0, 0) - U_Y^0 (1, \frac{1}{2})]}{\delta a} = \\
\frac{1}{2} \left( \frac{a}{2a-1} \right)^2 \left[ \frac{3a^2 - 4a + 1}{a^2} \log 2 + \frac{1}{2} \log \left( \frac{4+3p}{p} \right) \right] \\
> 0
\]
and the difference in expected utility provided by $(0,0)$ and by $(1,\frac{1}{2})$ monotonically increases with the productivity gap.

Notice further that it is:

\[
\lim_{a \to +\infty} U^Y_0 (0,0) - U^Y_0 (1,\frac{1}{2}) = \\
\frac{15}{8} \log 2 - \log \left(\frac{4-p}{2-p}\right) + \frac{1}{8} \left(\log \left(\frac{16+8p}{12+p}\right) + \log \left(\frac{4+2p}{4+p}\right)\right) \geq 0
\]

since for every $p \in (0,1]$, $\log \left(\frac{4-p}{2-p}\right) < \frac{15}{8} \log 2$ holds\(^5\).

A level for the productivity gap, $a^*$, such that the platform $(0,0)$ is preferred to $(1,\frac{1}{2})$ then, always exists. ■

Consider now the case where it is $p = 1$; there are no values $a \geq 1$ such that $U^Y_0 (0,0) - U^Y_0 (1,\frac{1}{2}) \leq 0$ since the following inequality holds:

\[
\left[U^Y_0 (0,0) - U^Y_0 \left(1,\frac{1}{2}\right)\right]_{a=1,p=1} = - \log \left(\frac{9}{8}\right) + \frac{1}{8} \left[\log \left(\frac{24}{13}\right) - \log \left(\frac{7}{4}\right) + \log \left(\frac{12}{5}\right)\right] > 0
\]

From previous observations it is possible to derive young agents preferred choice for different values of the productivity gap.

**Proposition 4** Young agents preferred platforms are:

- $(1,\frac{1}{2})$ if $1 < a \leq a^*$
- $(0,0)$ and $(1,0)$ if $a > a^*$

**Proof.** Any platform including $\theta = 1$ is not optimal since $U^Y_0 (\eta, 1) = -\infty$; this allows to discard the alternatives $(0,1)$ and $(1,1)$.

The comparison among $(0,0)$ (or equivalently $(1,0)$) and $(0,\frac{1}{2})$ reveals that young agents always prefer the first platform to the second since it is $U^Y_0 (0,0) - U^Y_0 (0,\frac{1}{2}) > 0$, i.e.:

\[
a - \frac{1}{4a} \log 2 + \frac{1}{4} \cdot \log \left(\frac{(2+p)^2}{3+p}\right) + \frac{1}{2} \cdot \log \left(\frac{8-4p}{4-p}\right) > 0
\]

A solution for the maximization problem of young workers then, is either $(0,0)$ or $(1,\frac{1}{2})$. Previous proposition shows that $U^Y_0 (0,0) \geq U^Y_0 (1,\frac{1}{2})$ when $a > a^*$; if this is the case, $(0,0)$ and $(1,0)$ are optimal choices.

If instead, it is $1 \leq a \leq a^*$, then $U^Y_0 (0,0) \leq U^Y_0 (1,\frac{1}{2})$ holds and this implies further that it is $U^Y_0 (0,\frac{1}{2}) \leq U^Y_0 (1,\frac{1}{2})$; the alternative $(1,\frac{1}{2})$ is young agents’ preferred platform. ■

\(^5\)Notice that the quantity $\left(\frac{4-p}{2-p}\right)$ monotonically increases with $p$; for $p = 1$ $\frac{15}{8} \log 2 < \log 3$ holds so that the condition $\log \left(\frac{4-p}{2-p}\right) < \frac{15}{8}$ is always verified.
The productivity gap drives young workers’ preferences.

When \( a \) is low, the selection process effectively increases expected income in the first period and guarantees insurance against the unemployment risk due to displacement; young agents then, choose a low degree of employment protection and postpone retirement.

Reducing retirement age to pursue the same scopes is more costly; the fiscal burden indeed, increases faster than the discounted value of future pension income, especially if the replacement rate is high.

If the productivity gap is large, the preferred level of employment protection is low and retirement age is high; this is the case because a small number of outsiders are hired through the selection process.

4.2.4 Voting stage equilibrium

The outcomes of the political process are described in the proposition that follows

**Proposition 5** If \( a > a^* \), the platforms \((1, 0)\) and \((0, 0)\) are Condorcet winners of the voting stage.

If \( 1 \leq a \leq a^* \), the platform \((1, \frac{1}{2})\) is a Condorcet winner of the voting stage.

**Proof.** Any platform including \( \theta = 1 \) is not a Condorcet winner of the voting stage since it is the case that \( U_{0}^{HP}(\eta, 1) = U_{0}^{LP}(\eta, 1) = U_{0}^{Y}(\eta, 1) = -\infty \); this allows to discard the alternatives \((0, 1)\) and \((1, 1)\).

The comparison between the platforms \((0, \frac{1}{2})\) and \((0, 0)\) (or equivalently to \((1, 0)\) given that agents are indifferent between them if \( \theta = 0 \)) reveals that the second alternative always gets a majority against first. When \( \eta = 0 \), high and low productivity old solve the same maximization problem and have identical preferences; if previous condition holds, it is \( \delta U_{0}^{LP} \frac{\partial}{\partial \eta} |_{\eta=0} = \delta U_{0}^{LP} \frac{\partial}{\partial \eta} |_{\eta=0} \leq 0 \) and both types of workers choose \((0, 0)\).

Given that proposition 11 shows that also young agents prefer \((0, 0)\) to \((0, \frac{1}{2})\), this last platform can be discarded.

Consider now the comparison between \((0, 0)\) (or \((1, 0)\)) and \((1, \frac{1}{2})\); notice that if the first alternative prevails against the second, it is also a Condorcet winner for the voting game.

Low productivity old workers always prefer the platform \((0, 0)\) to \((1, \frac{1}{2})\) since \( U_{0}^{LP}(0, 0) - U_{0}^{LP}(1, \frac{1}{2}) > 0 \) holds. It is in fact:

\[
\frac{1}{2} \log \left( \frac{2 + p}{1 + \frac{3}{4}} \right) > 0
\]

High productivity old and young workers choose \((0, 0)\) if it is \( U_{0}^{HP}(0, 0) \geq 0 \) and \( U_{0}^{Y}(0, 0) - U_{0}^{2}(1, \frac{1}{2}) \geq 0 \); this requires respectively that:

\[
\frac{1}{2} \log \left( \frac{4 + 2p}{3 + \frac{7}{4}} \right) = \frac{1}{2} \log \left( \frac{1}{2a - 1} \right) \log \left( \frac{1 + 3}{p + 4} \right) \geq 0
\]
and

\[
\frac{1}{2} \left[ 1 + \frac{(a - 1)^2}{a (2a - 1)} \right] \log 2 - \log \left( \frac{1 - \frac{p}{2}}{1 - \frac{p}{2}} \right) + \\
\frac{1}{4} \left\{ U_0^{HP} (0, 0) - U_0^{HP} \left( 1, \frac{1}{2} \right) \right\} + \left[ U_0^{LP} (0, 0) - U_0^{LP} \left( 1, \frac{1}{2} \right) \right] \geq 0
\]

hold. Last inequality implies that whenever young agents prefer the platform \((1, \frac{1}{2})\) to \((0, 0)\) the same is true also for high productivity old workers since

\[
\frac{1}{2} \left[ 3 + \frac{(a - 1)^2}{a (2a - 1)} \right] \log 2 - \log \left( \frac{4 - p}{2 - p} \right) + \frac{1}{4} \left[ U_0^{LP} (0, 0) - U_0^{LP} \left( 1, \frac{1}{2} \right) \right] > 0
\]

holds; substituting for \(U_0^{LP} (0, 0)\) and \(U_0^{LP} \left( 1, \frac{1}{2} \right)\) gives indeed:

\[
\frac{13}{8} \log 2 - \log \left( \frac{4 - p}{2 - p} \right) + \frac{(a - 1)^2}{a (2a - 1)} \log 2 + \frac{1}{8} \log \left( \frac{2 + p}{4 + p} \right) > 0
\]

Previous quantity is always positive given that for every \(p \in (0, 1]\), it is \(\frac{13}{8} \log 2 > \log \left( \frac{4 - p}{2 - p} \right)^6\).

Notice that when \(a > a^*\), \(U_Y^Y (0, 0) > U_Y^Y \left( 1, \frac{1}{2} \right)\) holds and the platforms \((0, 0)\) and \((1, 0)\) are Condorcet winners of the voting game; young and low productivity old workers form the majority that supports this outcome.

When \(1 \leq a \leq a^*\) and \(U_Y^Y (0, 0) \leq U_Y^Y \left( 1, \frac{1}{2} \right)\) hold, it is also \(U_0^{HP} (0, 0) < U_0^{HP} \left( 1, \frac{1}{2} \right)\); the platform \((1, \frac{1}{2})\), supported by young and high productivity old workers, prevails over \((0, 0)\).

Given that all agents prefer \((0, 0)\) to \((0, \frac{1}{2})\), young workers and high productivity old choose \((1, \frac{1}{2})\) when compared with \((0, \frac{1}{2})\); the first platform then is preferred to all the others and is the Condorcet winner of the voting game.

Indifference over the degree of employment protection when \(\theta = 0\) implies that the platforms \((0, 0)\) and \((1, 0)\) are both Condorcet winners for the voting stage when \(a > a^*\); considering agents’ preferences when \(\theta\) approaches zero permits to restrict the set of equilibria for the political process.

**Proposition 6** When \(a > a^*\) and agents’ preferences in a neighborhood of \(\theta = 0\) are considered, it is possible to find a value for the productivity gap, \(a^{**}\), such that:

- the platform \((0, 0)\) is a Condorcet winner for the voting stage if \(a \geq a^{**}\)
- the platform \((1, 0)\) is a Condorcet winner for the voting stage if \(a^{**} < a\)

Notice that the quantity \(\left( \frac{4 - p}{2 - p} \right)\) monotonically increases with \(p\); for \(p = 1\) \(\frac{13}{8} \log 2 < \log 3\) holds so that the condition \(\log \left( \frac{4 - p}{2 - p} \right) < \frac{13}{8}\) is always verified.
Proof. Low and high productivity old workers have opposite preferences with respect to the degree of employment protection $\eta$, when $\theta$ approaches zero. The first type of agents chooses $\eta = 1$ since, for every $\theta > 1$, $\frac{MU_Y^{HP}(\eta, \theta)}{\eta} > 0$ holds; on the contrary low productivity old prefer $\eta = 0$ since it is $\frac{MU_Y^{LP}(\eta, \theta)}{\eta} < 0$.

Young workers are pivotal in the choice between the alternatives $(0, 0)$ and $(1, 0)$; consider the following transformation of the first derivative of expected utility with respect to $\eta$, for these agents:

$$\frac{MU_Y^{PS}}{\eta} = \frac{1}{4} \left( \frac{a - 1}{a} \right) \left( \frac{1}{2a - 1} \right) \log \left[ 1 + \theta \cdot \frac{1 - \frac{1}{p}(1 - \theta)}{1 + \frac{1}{p}(1 - \theta)} \right] +$$

$$+ \frac{1}{8} \left( \frac{1}{2a - 1} \right) \log \left[ 1 + \theta \cdot \frac{1 - \frac{1}{p}(1 - \theta)}{1 + \frac{1}{p}(1 - \theta)} \right] - \frac{1}{8}$$

Take the limit for $\theta$ that approaches 0 to get:

$$\lim_{\theta \to 0} \frac{MU_Y^{PS}}{\eta} = \frac{1}{4} \left( \frac{a - 1}{a} \right) \left( \frac{2 + p}{2 - p} \right) +$$

$$+ \frac{1}{8} \left( \frac{2 - p}{p} \right) - \frac{1}{8}$$

In order for $\eta = 0$ to be preferred over $\eta = 1$ previous quantity must be negative and this requires further:

$$\phi(a, p) = \left( \frac{a - 1}{a} \right) \left( \frac{2 + p}{2 - p} \right) + \frac{1}{p} - a < 0$$

Notice that since $\lim_{a \to +\infty} \phi(a, p) = -\infty$ and $\phi(1, p) = \frac{1}{p} - 1 \geq 0$ hold, solving for the biggest value for $a$ such that $\phi(a, p) = 0$ defines the condition for $(0, 0)$ to be preferred over $(1, 0)$; in particular, it is the case that young agents choose the first alternative when $a > a^*$, where $a^*$ is:

$$a^* = \frac{1}{2} \left[ \left( \frac{2 + p}{2 - p} + \frac{1}{p} \right) + \sqrt{\left( \frac{2 + p}{2 - p} + \frac{1}{p} \right)^2 - 4 \left( \frac{2 + p}{2 - p} \right)} \right]$$

If the productivity gap exceeds previous threshold, the alternative $(0, 0)$ is a Condorcet winner; if $a \leq a^*$, the platform $(1, 0)$ is the equilibrium outcome for the voting stage. 

A further observation is worthy with respect to the comparison between the thresholds $a^*$ and $a^{**}$; it is the case that $a \geq a^*$ does not imply $a > a^{**}$.

Consider for instance the case where $a = 1$ and $p$ approaches 1; it is the case that $\lim_{p \to 1} U_0^Y (0, 0) - U_0^Y (1, \frac{1}{4}) > 0$ holds while $\lim_{p \to 1} \phi(1, p)$ approaches zero from below. It is indeed $\phi(1, p) = \frac{1}{p} - 1$. 

19
In a neighborhood of \( p = 1 \) then, it is \( a^{**} \geq 1 > a^* \).

A large productivity gap is required to have high employment protection and early retirement; as the value of \( a \) decreases the equilibrium setup for the Social Security System entails lower employment protection and later retirement.

5 Employment Protection and Retirement Age: a Cross-Country Analysis.

In the theoretical model a positive correlation is found between the degree of employment protection and the size of early retirement; the productivity gap is the variable that drives the choice over these parameters and causes different setups for the Social Security System to emerge in equilibrium.

The empirical analysis is aimed to test previous result and to provide an explanation for the observed differences in the institutional frameworks of real economies; this is done in the same spirit of Kristov et al. (1992), Breyer and Craig (1995), and more recently Persson (2002).

5.1 Data Description

Institutions are slow moving mechanisms and a long period is required before tangible reforms are implemented through the political process; the empirical analysis then, should consider a wide time span to account for these changes but long data series for relevant variables are not easily available. Information scarcity is further exacerbated because the setup of the Social Security System is defined at the national level and each country provides only one observation per period.

Due to previous restraints, the dataset used for the estimates is a small one; the unbalanced panel includes five year average observations over a set of 22 countries\(^7\). The considered period goes from 1980 to 1998 (for the interval 1995-1998 a four year average is used) and every statistical unit has a maximum of four observations.

Current analysis mostly uses statistics from the OECD Databases; other informations on specific issues are retrieved from two main sources: the Barro-Lee Database\(^8\) and the Penn World Tables.

Employment protection and retirement age are the dependent variables in the estimates.

The employment protection index (EPI), calculated by the OECD, measures the first quantity; this index is the result of a procedure that assigns different scores to 18 dimensions of the employment protection legislation, i.e. the set of rules adopted in firing and hiring workers.

\(^7\)The countries in the dataset are: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Poland, Slovak Republic, Spain, Sweden, Switzerland, Turkey, U.K. and U.S.A.

\(^8\)See Barro and Lee (2000) for a description of the data.
There are two versions of the EPI: the first one measures the protection of regular and temporary employees, the second includes also the regulation of collective dismissals. The ranking of the OECD countries does not change significantly considering one index or the other; the first version then, is used since a longer time series is available9.

The standard age of entitlements to public pension in OECD economies, ranges from 60 to 67 for men and from 59 to 67 for women. Many countries allow early retirement and workers can opt for a reduced pension before the standard age; Conde-Ruiz and Galasso (2003) and Gruber and Wise (1999) moreover, point out that exits to retirement are made available also by legislation on disability and on unemployment benefits.

A complex bunch of provisions defines the moment in time when a worker is entitled of a full or a reduced pension; a measurement problem then, arises with respect to the definition of the actual retirement age. Present work uses a measure of early retirement for that.

The early retirement index (ERI) calculated by DICE at CESifo is the second dependent variable in the estimates; the ERI is defined as 100% minus the participation rate for men aged 55-64.

Previous index describes the behavior of a paradigmatic worker and does not exactly meet the requirements of current analysis; retirement legislation though, is hardly disentangled and it is extremely difficult to calculate measures based on institutional parameters. The ERI is considered as a proxy for this missing information given that countries where the Social Security System provides more and better options for early retirement should be also those where people in fact retire earlier.

The theoretical model identifies three main explanatory variables.

The first one is the productivity gap between different workers; this quantity is not observed in the real world, therefore the efficiency of the schooling system is used as a proxy for it.

The share of the population aged 25 and over that completed at least the second level of education (Atleastseccompleted), is included in the estimates; data come from the Barro and Lee (2000) database. Under the hypothesis that labor productivity is an increasing function of education and that marginal productivity is decreasing, if many people attain a degree that exceeds the mandatory primary level, the average gap between workers is going to be low.

The second explanatory variable is electorate composition; the number of high and low productivity agents in each generation defines the probability to be a high type worker. Consider how an increase in this variable affects the preferences of the pivotal voters i.e. of the young.

A first effect is that the unemployment risk is reduced and consequently the support for high employment protection decreases. On the other hand if the selection process involves a small number of workers, total turnover is low; early retirement is required to increase outsiders’ entries in the employment pool.

The number of high productivity agents in the electorate is measured by

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9 For a detailed description of the index see OECD (2004).
the share of the population aged 25 and over, with a post-secondary level of education completed (postseccompleted); also in this case education is used as a proxy for labor productivity. Data come from the Barro-Lee dataset.

The last regressor is the generosity of the pension system (costbenefratio); this variable captures the effect of variations in the replacement rate levels that characterize different countries. Consider how this variable affects young agents’ preferences.

A generous pension system provides a high income in the old age but requires also high contributions; in this context, early retirement is beneficial for the young generation because it anticipates outsiders’ entries into employment and at the same time increases taxation on wages. The combination of these effects realizes a balanced resource allocation over the two periods.

Employment protection is used only to reduce unemployment risk if the agent is a low productivity type; the effect is positive both on employment protection and on early retirement.

The variable that measures the generosity of the pension system is the ratio of government revenues classified as social security contributions to public expenditures for old age pensions; data on pension expenditures are retrieved from the OECD Social Expenditures Database while those on social security contributions come from the OECD Revenue Statistics.

Some controls are included in the estimates. The share of self-employment on total (selfempl) accounts for the preferences of workers that are not directly affected by the setup of the Social Security System; data are retrieved from OECD (2006).

A second control is labor demand increase measured by gross per capita GDP growth (gdpgrowth); a higher demand enhances outsiders’ entries into employment and works as a decrease in the average productivity gap. The Penn World Table Database provides data for this variable.

A measure of capital stock variation is also included (investmentshareofgdp); a high level of investments fosters technological change and increases the gap between high and low productivity workers. The investment share of total GDP reported by the Penn World Table Database is used in the estimates.

The analysis controls for labor supply changes and in particular, for inflows of non-resident workers (migrationrate); this variable is measured by the migration rate and is equivalent to an increase in the productivity gap. Data come from OECD Population and Vital Statistics Database.

5.2 Specification

In the theoretical model employment protection and retirement age are jointly determined by the political process; the empirical specification thus, relies on the system of simultaneous equation described below:

\[
\begin{align*}
    y_{1t}^1 &= \alpha^1 y_{1t}^2 + \beta^1 X + \gamma^1 Z + \epsilon_{1t}^1 \\
    y_{2t}^2 &= \alpha^2 y_{1t}^1 + \beta^2 X + \gamma^2 Z + \epsilon_{2t}^2
\end{align*}
\]
where \( y_{1i} \) and \( y_{2i} \) denote respectively the EPI and the ERI index, \( X \) is the matrix of explanatory variables and \( Z \) is the matrix of control variables; the error term includes two elements: a country specific time invariant error (\( e_{1i} \) and \( e_{2i} \)) and an idiosyncratic error (\( \varepsilon_{1i} \) and \( \varepsilon_{2i} \)).

The system is estimated using instrumental variables; both the Balestra and Varadharajan-Krisnakumar (1987) generalized two-stage least square and the Baltagi (1981) error component two-stage least square estimators were calculated. The results obtained with the first procedure are reported since there are no significant differences between the two.

The social security contributions share of GDP, lagged one-period, is the instrument for \( y_{1i} \) and \( y_{2i} \).

If model predictions are correct, the dependent variables vary together; when the value of the first is high, the same is true also for the second. Large pension expenditures due to a high level of early retirement require proportional payments to the social security system; workers’ contributions then, are expected to be highly correlated with both ERI and EPI and at the same time to be uncorrelated with the idiosyncratic error terms.

Random effects and fixed effects estimators have been computed for the two equations; the Hausman specification test pointed out that a random effects model is preferable in the context of present analysis.

5.3 Results

Current section presents the estimates of the random effects model for the system of equation described above; preliminary to that some test of correct specification are reported.

A Davidson and MacKinnon (1993) test is conducted to check wether or not there is exogeneity among the dependent variables; if the null hypothesis that an ordinary least square regression gives consistent estimates is rejected, a system of simultaneous equations describes the data generating process and the use of instrumental variables is justified.

A second control relies on the standard Hausman test and is intended to choose between the fixed and the random effects specification.

Consider the first equation where the employment protection index is regressed against the early retirement index and the set of explanatory and control variables.
The F statistics for the exogeneity test has a p-value amounting to 0.88 and does not allow to reject the null hypothesis. The Hausman specification test could not be conducted since the variance/covariance matrix for the coefficients of the random effects estimates turned out to be bigger than that obtained through a fixed effects estimate; the small size of the sample is likely to be the main cause for this statistical problem.

The estimates for the second equation where the early retirement index is regressed against the employment protection index returned the following results:

| epi          | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|--------------|-------|-----------|-------|------|---------------------|
| epi          | 1.911538 | 0.096618 | 2.66 | 0.008 | 1.724090 – 2.099058 |
| postsecomact | 2.432652 | 1.37045 | 1.78 | 0.040 | 0.726299 – 3.138901 |
| attsecomact   | -0.379736 | 2.37146 | -0.16 | 0.871 | -8.307831 – 7.558359 |
| combenefit    | -4.228628 | 4.087602 | -1.03 | 0.301 | -12.24013 – 3.782973 |
| selfempl      | -7.30021 | 35.70166 | -0.20 | 0.840 | -141.3046 – 137.7044 |
| gdpgrowth     | 1.577812 | 1.652013 | 0.95 | 0.344 | -1.683556 – 4.832189 |
| investmen      | -4.991254 | 4.993087 | -0.90 | 0.370 | -13.95793 – 4.074427 |
| migrationpm   | -6.664656 | 1.174374 | 5.67 | 0.000 | -13.30578 – 0.00000 |
| _cons         | 11.144811 | 28.888812 | 0.39 | 0.699 | -45.4440 – 67.73361 |
The p-value for the F statistics from the Davidson-MacKinnon test is 0.0129; the null hypothesis is not accepted for a 5% interval of confidence. The Hausman test does not allow to reject the null that random effects and fixed effects estimators are equivalent; using a common estimate for the variance/covariance matrix of the coefficients based on the most efficient model indeed, produces a chi-squared statistic of 0.54 (Prob > chi2 = 0.99).

Previous specification is supported by some empirical evidence; the endogeneity of the dependent variables is clear at least in the second equation. The choice of the random effects model moreover, finds a partial justification in the data especially when the second equation is considered; this result is coherent with the findings of Breyer and Craig (1995) and with the nature of the phenomenon under analysis.

Notice that the time component of the data has little explicative power; the variance between is always bigger than the variance within countries; in particular, in the second estimate, the first quantity is two times the second. The fixed effects play a significant role and cannot be discarded without incurring in a huge loss of information.

Dealing with standard variables and index calculated by international institutions moreover, reduces the probability that measurement errors due to country-specific classification methods cause correlations between the regressors and the time-invariant error term. Substantial arguments then, exist for the use of a random effects model.

Consider now the results of the analysis.

In the equation where employment protection is regressed against early retirement, this last variable is not significant for a 10% confidence interval but has the positive sign predicted by the theoretical model. All the coefficients of the explanatory variables have the expected sign and two out of three are also statistically significant. The fraction of high productivity workers and the share of the population aged 25 and over that completed at least the second level of education are significant respectively for a 10% and for a 5% confidence interval and have a negative effect on the dependent variable as predicted by the model; the regressor costbenfrtsratio however, is not significant.

No control variables are significant.

In the second equation where the early retirement index is regressed against the employment protection index, the variable epi has a highly significant positive coefficient. Among the explicative variables only one out of three is significant and has the expected sign; electorate composition indeed, is significant for a 10% confidence interval and has a positive coefficient.

The variable Atleastseccompleted has the correct negative sign but is not statistically significant; the generosity of the pension system is not significant as well and also the sign of the coefficient is not the expected one.

The fraction of self-employment on total is the only significant control variable; it has a negative impact on early retirement and is statistically significant for a 5% confidence interval.
Estimates results show that there is some evidence in the data of a direct relationship between employment protection and retirement age; in particular, labor market regulation affects significantly the setup of the pension system.

Two variables mostly influence the interaction between the institutional parameters under analysis: electorate composition and the productivity gap; the first element is important for both retirement age and employment protection while the effects of the second are concentrated on the last aspect. Furthermore all the explanatory variables, with a unique exception, have the expected sign.

Previous observations support the description of the economy presented in the model even if no unquestionable confirmation is found in the data.

6 Final Remarks

The present paper contributes to current debate on Social Security System by analyzing the mutual interdependence between labor market and pension system; in particular, the joint choices on employment protection and on mandatory retirement age are considered.

The young generation is pivotal in the voting. An optimal solution for these agents depends on two elements: income allocation across periods and across states of the world; since no savings means nor insurance devices against unemployment are available, the Social Security System is used to pursue these scopes.

When the productivity gap is high, a low level of employment protection is not effective in transferring resources from future to current period; moreover, it does not guarantees a significant increase in the probability of getting a new job when an agent is displaced and reveals to be a high productivity type. Early retirement is chosen to enhance entries in the labor market during the first period and to increase current income; complete employment protection is implemented to fully insure against the state of the world where the agent is a low type.

If the productivity gap is small, the efficacy of the selection process rises; this enhances the opportunities to transfer income from the second period to the first one and reduces significantly the risk of unemployment due to displacement. As a consequence the favor of the young generation for low protection and late retirement increases.

Under the hypothesis of logarithmic utility, the political process produces three main outcomes.

A first one is characterized by no employment protection and late retirement and emerges when the productivity gap is small; as this variable rises, retirement age decreases and early retirement is implemented together with no protection. Further increases in the productivity gap define a setup for the Social Security System that includes complete employment protection and early retirement.

A weak direct relationship is found between employment protection and early retirement; what causes these elements to vary together is the size of the
productivity gap; a support to this prediction of the model is found in the data but there is no clear-cut evidence for it.

The empirical analysis indeed, meets with some obstacles. The first one is data scarcity that does not allow to have long time series on institutional variables; the second one is the difficulty to find a reliable measure of the productivity gap that is robust both for the estimates concerning the pension system and for that concerning the labor market.

Despite these shortcomings the present works provides some useful contributions to the debate on Social Security System reforms. Total labor market turnover turns out to be a crucial factor in this context; any amendment to pension system or employment protection regulation is constrained by this variable if a political consensus is required for its implementation.

A change in the mandatory retirement age that is not coupled with a coherent reform of the labor market then, risk to be unpopular and politically unfeasible; the same problem arises with respect to interventions that affect employment protection regulation and ignore the timing of retirement.

7 Appendix 1: Equilibrium for the Representative Firm Maximization Problem

Consider initially a description of the subgames where the representative firm moves.

Right after the political process, the quantity \( x_1^1 \) is chosen and a strategy for the firm \( \Sigma_1^F \) is a function of its initial stock of labor, \( L_1^0 \), of the decrease in input due to displacement, \( \gamma_1^0 \) and of the technology parameter:

\[
\Sigma_1^F \left( L_1^0, \gamma_1^0, \alpha \right) : [0, \frac{1}{\sigma} (a + 1)] \times [0, \frac{1}{\sigma} (a + 1)] \times [0, 1] \rightarrow [0, (a + 1)]
\]

Notice that both \( L_1^0 \) and \( \gamma_1^0 \) never exceed \( \frac{1}{\sigma} (a + 1) \), i.e. total labor provided by one generation, since at the beginning of each period only old agents are employed.

In the third stage, the representative firm chooses \( x_2^2 \); a strategy, \( \Sigma_2^F \), is a function of previous subperiod stock of labor, \( L_2^0 \), of input reduction due to labor selection, \( \gamma_2^0 \), and of the technology parameter:

\[
\Sigma_2^F \left( L_2^0, \gamma_2^0, \alpha \right) : [0, (a + 1)] \times [0, 1] \times [0, 1] \rightarrow [0, (a + 1)]
\]

When all old workers retire, the representative firm gets from the market the quantity \( x_3^0 \) of labor; a strategy \( \Sigma_3^F \), is a function of the stock of input in the period going from date \( \frac{1}{\theta} \) to date \( \frac{1}{\theta} \cdot \frac{1}{\sigma} (a + 1) \), \( L_3^0 \), of labor decrease due to old workers retirement, \( \gamma_3^0 \), and of the technology parameter:

\[
\Sigma_3^F \left( L_3^0, \gamma_3^0, \alpha \right) : [0, (a + 1)] \times [0, \frac{1}{\sigma} (a + 1)] \times [0, 1] \rightarrow [0, (a + 1)]
\]

Notice that the hiring decisions described above, change, each period, the composition of employment and unemployment; this happens both with respect to agents’ type (high or low productivity) and to agent’s age (old or young) and implies that also the size of \( \gamma_1^0 \), \( \gamma_2^0 \) and \( \gamma_3^0 \) are affected by the actions of the firm.

Despite it is assumed that high productivity workers get hired first, different workers are perfectly fungible. The representative firm is indifferent with respect
to the composition of labor input; since there are no hiring nor firing costs, also the size of the flows is irrelevant. In order to simplify the analysis, it is possible then, to consider the quantities $\gamma_1$, $\gamma_2$ and $\gamma_3$ as exogenous parameters.

7.1 Retirement Stage

At time $t + \theta_2$ old workers retire and the representative firm chooses the quantity $x_0^3$ of labor to get from the market; its maximization problem is the following:

$$V_0^3 = \max_{x_0^3} (L_0^3 - \gamma_0^3 + x_0^3)^\alpha - W_0^3 (L_0^2 - \gamma_0^2 + x_0^2) + \frac{1}{2} \cdot V_1^1 [L_0^3 (x_0^3), x_1]$$

where $W_0^3$ is current price of one unit of efficient labor and $L_0^2$ is defined as follows:

$$L_0^3 = L_0^2 - \gamma_0^2 + x_0^2$$

Future profits, $V_1^1 [L_0^3 (x_0^3), x_1]$, are discounted at the same rate used for the maximization problem of workers.

The first order condition with respect to $x_0^3$ is:

$$\frac{\delta V_0^3}{\delta x_0^3} = \frac{\alpha}{(L_0^3 - \gamma_0^3 + x_0^3)^{1-\alpha}} - W_0^3 + \frac{1}{2} \cdot \frac{\delta V_1^1}{\delta x_0^3} = 0$$

7.2 Selection Stage

At time $t + \theta_2$ the representative firm can fire a fraction $\eta$ of low productivity workers; the stock of labor then, decreases of the quantity $\gamma_0^2$.

The maximization problem for the representative firm is the following:

$$V_0^2 = \max_{x_0^2} (L_0^2 - \gamma_0^2 + x_0^2)^\alpha - W_0^2 (L_0^1 - \gamma_0^1 + x_0^1) + V_0^3 [L_0^2 (x_0^2), x_0^3]$$

where $W_0^2$ is current price of one unit of efficient labor and $L_0^1$ is:

$$L_0^2 = L_0^1 - \gamma_0^1 + x_0^1$$

The first order condition with respect to $x_0^2$ is:

$$\frac{\delta V_0^2}{\delta x_0^2} = \frac{\alpha}{(L_0^2 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^2 + \frac{\delta V_0^3}{\delta x_0^2} = 0$$

From the envelope theorem it is the case that:

$$\frac{\delta V_0^3}{\delta x_0^2} = \frac{\alpha}{(L_0^2 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^3$$

Substituting the above equation in the first order condition gives:
\[
\frac{\delta V_0^2}{\delta x_0^2} = \frac{\alpha}{(L_0^2 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^2 + \frac{\alpha}{(L_0^3 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^3 = 0
\]

7.3 Displacement

At time 0 the displacement process causes a reduction in labor input amounting to \(\gamma_1^0\); the representative firm solves the following maximization problem:

\[
V_1^0 = \max_{x_1^0} \left( (L_0^1 - \gamma_0^1 + x_1^0)^\alpha - W_0^1 (L_0^1 - \gamma_0^1 + x_1^0) + V_0^2 [L_0^2 (x_1^0), x_0^2] \right)
\]

where \(W_0^1\) is the current price of one unit of efficient labor and \(L_0^1\) is:

\[
L_0^1 = L_3^1 - \gamma_1^1 + x_1^1
\]

The first order condition with respect to \(x_0^1\) is:

\[
\frac{\delta V_0^1}{\delta x_0^1} = \frac{\alpha}{(L_0^2 - \gamma_0^1 + x_0^2)^{1-\alpha}} - W_0^1 + \frac{\delta V_0^2}{\delta x_0^2} = 0
\]

From the envelope theorem is:

\[
\frac{\delta V_0^2}{\delta x_0^2} = \frac{\alpha}{(L_0^2 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^2
\]

Substituting the above equation in the first order condition gives:

\[
\frac{\delta V_0^1}{\delta x_0^1} = \frac{\alpha}{(L_0^1 - \gamma_0^1 + x_1^1)^{1-\alpha}} - W_0^1 + \frac{\alpha}{(L_0^3 - \gamma_0^1 + x_0^2)^{1-\alpha}} - W_0^3 = 0
\]

7.4 The Choice of the Firm: Equilibrium

From the maximization problem of the displacement stage is possible to derive, through the envelope theorem, the quantity \(\frac{\delta V_0^1}{\delta x_0^1}\):

\[
\frac{\delta V_0^1}{\delta x_0^1} = \frac{\alpha}{(L_1^1 - \gamma_1^1 + x_1^1)^{1-\alpha}} - W_1^1
\]

and rewrite the first order condition relative to the retirement stage as:

\[
\frac{\delta V_0^3}{\delta x_0^3} = \frac{\alpha}{(L_3^1 - \gamma_0^2 + x_0^2)^{1-\alpha}} - W_0^3 + \frac{1}{2} \left[ \frac{\alpha}{(L_1^1 - \gamma_1^1 + x_1^1)^{1-\alpha}} - W_1^1 \right] = 0
\]
Proposition 7 In equilibrium, the representative firm holds constant the level of labor input at \( \frac{1}{2} (a + 1) \). Labor market is either in equilibrium or faces excess supply and each unit of efficient labor is paid \( W_{LP} \).

Proof. Guess that labor employed in the production process is always \( \frac{1}{2} (a + 1) \) and that every subperiod, the price for each unit of efficient labor is \( W_{LP} \).

Verify that the first order conditions are fulfilled starting from the last stage:

\[
\frac{\delta V_3}{\delta x_3} = 0
\]

for the remaining two it is the case that:

\[
\frac{\delta V_1}{\delta x_1} = \frac{\alpha}{\left[ \frac{1}{2} (a + 1) \right]^{1-\alpha}} - \frac{\alpha}{\left[ \frac{1}{2} (a + 1) \right]^{1-\alpha}} = 0
\]

\[
\frac{\delta V_2}{\delta x_2} = \frac{\alpha}{\left[ \frac{1}{2} (a + 1) \right]^{1-\alpha}} - \frac{\alpha}{\left[ \frac{1}{2} (a + 1) \right]^{1-\alpha}} = 0
\]

Check now that on the labor market, there is either excess supply or equilibrium, if an efficient unit of labor is paid \( W_{LP} \).

Notice that when all old workers retire, a complete turnover happens and the whole young generation gets hired by the representative firm; at the beginning of the period, all the old are employed while all the young are unemployed and labor supply exceeds the demand.

The displacement process causes a reduction in input amounting to:

\[ \gamma_1^1 = \frac{1}{4} (a + 1) \]

this is the case because the employment pool includes in the same proportion high and low productivity workers. Moreover, since the representative firms keeps constant the level of input, it is the case that:

\[ x_1^1 = \gamma_1^1 \leq \frac{1}{2} (a + 1) \]

and labor market faces excess supply since the unemployment pool includes, the whole young generation. The representative firm then, hires only high productivity workers; indeed it is \( \frac{1}{4} a \geq \gamma_1^1 \).

Before the selection process, the number of low productivity workers employed in the representative firm is \( \frac{1}{4} \); this means that labor input is reduced by the quantity:

\[ \gamma_2^1 = \frac{\eta}{4} = x_2^1 \leq \frac{1}{2} (a + 1) \]

and again labor supply exceeds labor demand.
In the last stage the old retire and the expected reduction in labor input amounts to:

\[ \gamma_t^3 = \frac{a}{4} + \frac{1}{4} \left( 1 - \frac{\eta}{2} \right) + \frac{\eta}{4} = x_t^3 \]

Since the representative firm is an approximation for the whole economy, many workers are involved in the process; the law of large numbers then, guarantees that actual and expected values of \( \gamma_t^3 \) coincide.

All the unemployed young are hired at this point and marginal productivity of one unit of efficient labor is equal to the minimum wage; the market is in equilibrium.

8 Appendix 2: Representative Firm Preference for High Productivity Workers

This section is aimed to give a formal justification to the assumption that high productivity workers are preferred by the representative firm.

Consider a setup where hiring a high productivity workers reduces production costs; representative firm profits are defined as follows:

\[
\Pi = Y_t - W_{t+1}^{LP} \cdot L_{t+1}^{LP} + W_t^{HP} \cdot L_t^{HP} + \frac{1}{2} \left( Y_{t+1} - W_{t+1}^{LP} \cdot L_{t+1}^{LP} + W_{t+1}^{HP} \cdot L_{t+1}^{HP} \right) + i \cdot L_t^{HP} + \frac{1}{2} \left( Y_{t+1} - W_{t+1}^{LP} \cdot L_{t+1}^{LP} + W_{t+1}^{HP} \cdot L_{t+1}^{HP} \right)
\]

where the parameter \( i \) describes the decrease in costs produced by high productivity workers \((HP)\).

Assume now that \( i \) is small enough to have excess supply for these agents when both generations are present on the labor market; in other words for \( a \geq 1 \)

\[ \alpha \cdot a^a + i \leq \frac{a \cdot \alpha}{\left[ \frac{1}{2} (a + 1) \right]^{1-\alpha}} \]

must hold and marginal productivity of \( HP \) agents is smaller than their minimum wage when they are the only employees in the firm.

The representative firm prefers to hire \( HP \) workers because, for a given stock of input, each unit of efficient labor from these agents generates a marginal benefit that exceeds that provided by low productivity workers \((LP)\) by the factor \( \frac{1}{2} \) and has the same cost; also the market for \( LP \) agents indeed, faces excess supply when both generations are present on the labor market.

Consider now the decision of the representative firm when labor supply shrinks due to retirement.

Suppose that, even if it is \( \theta = 1 \), agents do not work until they die but there is a period of length \( \varepsilon > 0 \) where the old generation exits from the labor market. When this happens, the representative firm decides how many high and low productivity workers to hire; the choice of the firm defines workforce size and composition at the beginning of the next period.
When old individuals retire, labor market faces excess demand for HP agents; even if all workers were hired in fact, it would be:

\[
\frac{a \cdot \alpha}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}} + i > \frac{a \cdot \alpha}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}}
\]

All HP agents then, get hired and the wage ratio equals their marginal productivity.

The demand for LP workers instead, either exceeds or is equal to the supply at the minimum wage. Hiring low productivity workers in this context, is beneficial both because it increases overall production and because reduces HP wage rate; on the other hand each LP agent employed limits the possibility to exploit the reduction in costs produced by HP workers up until the selection process (at least for those individuals that are not hit by the displacement process).

Consider now, how many of these agents get hired; it is possible to show that if \( i \) is small enough, all the low productivity workers get a job.

In order to simplify the analysis, a "worst case scenario" is considered where LP individuals employed at the beginning of the period, work in the representative firm until mandatory age set at \( \theta = 1 \); this amounts to say that there is no displacement and no selection. At time 0 then, labor input derived from the previous period is unchanged (in terms of size and composition) until retirement unless the firm hires new workers.

The maximization problem of the firm when the old generation retires is the following:

\[
\begin{align*}
\text{Max}_{L_{t}^{LP},L_{t}^{HP}} & \cdot \left\{ \left( \frac{1}{2} \cdot a + L_{t}^{LP} \right)^{\alpha} - \frac{\alpha}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}} \cdot L_{t}^{LP} - \frac{a}{2} \cdot \frac{\alpha}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}} \cdot L_{t}^{LP} + \frac{i}{2} \cdot a + L_{t}^{HP} \right\} + \\
+ & \frac{1}{2} \left\{ \frac{\alpha \cdot a}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}} \cdot L_{t}^{HP} + \frac{i}{2} \cdot a + L_{t}^{HP} \right\}
\end{align*}
\]

Notice that the initial level of labor input is set at \( \frac{1}{2} \cdot a \) since all high productivity workers are hired after retirement.

Consider the first order conditions for the maximization problem and start with that referred to \( L_{t+1}^{HP} \):

\[
\frac{1}{2} \left\{ \frac{\alpha \cdot a}{\left( \frac{1}{2} \cdot a + L_{t}^{LP} + a \cdot L_{t+1}^{HP} \right)^{1-\alpha}} - \frac{\alpha \cdot a}{\left[ \frac{1}{2} \cdot (a + 1) \right]^{1-\alpha}} + i \right\} = 0
\]

Deriving with respect to \( L_{t}^{LP} \) gives:
\[
\varepsilon \left\{ \left( \frac{1}{2} \cdot a + L_t^{LP} \right)^{1-\alpha} - \left( \frac{1}{2} (a + 1) \right)^{1-\alpha} + \frac{1}{2} \left( \frac{1}{2} \cdot a + L_t^{LP} \right)^{2(1-\alpha)} \right\} \\
+ \frac{1}{2} \left\{ \left( \frac{1}{2} \cdot a + L_t^{LP} + a \cdot L_t^{HP} \right)^{1-\alpha} - \left( \frac{1}{2} (a + 1) \right)^{1-\alpha} \right\} = 0
\]

Notice that from the first equation, it is the case that:

\[
\frac{\alpha}{\left( \frac{1}{2} \cdot a + L_t^{LP} + a \cdot L_t^{HP} \right)^{1-\alpha}} - \frac{\alpha}{\left( \frac{1}{2} (a + 1) \right)^{1-\alpha}} = \frac{1}{2} \cdot i
\]

Substituting this result in the second equation gives:

\[
\varepsilon \left\{ \left( \frac{1}{2} \cdot a + L_t^{LP} \right)^{1-\alpha} - \left( \frac{1}{2} (a + 1) \right)^{1-\alpha} + \frac{1}{2} \left( \frac{1}{2} \cdot a + L_t^{LP} \right)^{2(1-\alpha)} \right\} = \frac{1}{2} \cdot i
\]

In order to have \( L_t^{LP} = \frac{1}{2} \) it must be the case that:

\[
\varepsilon \cdot \frac{a}{2} \cdot \frac{\alpha (1-\alpha)}{\left( \frac{1}{2} (a + 1) \right)^{2(1-\alpha)}} - \frac{1}{2} \cdot i \geq 0
\]

holds; the above condition is always true if \( i \) approaches zero.

Assuming that the reduction in costs generated by \( HP \) workers is negligible then, implies that a complete generational turnover happens on the labor market after retirement. Previous result is obtained when no substitution of \( LP \) workers is possible and retirement age is set at its maximum; the same must happen \textit{a fortiori} when displacement and selection allow to replace some low productivity individuals and \( \theta \) is less than 1.

Consider now, that if at the beginning of each period the whole old generation is employed, displacement causes a total outflow of labor amounting to \( \frac{a}{2} (a + 1) \). This decrease in input is completely compensated if only high productivity young workers are hired; overall labor they provide indeed, is \( \frac{a}{2} \geq \frac{1}{2} (a + 1) \).

The demand for high productivity workers is less than total supply and each unit of labor provided by \( HP \) workers is paid \( \frac{\alpha}{\left[ \frac{1}{2} (a + 1) \right]^{2(1-\alpha)}} \); after displacement then, the representative firm prefers high productivity workers to \( LP \) agents given that for the same cost, they provide the additional benefit \( \frac{a}{\alpha} \). An analogous argument applies also to the selection stage.

The assumption that \( HP \) workers are hired first can be thought as an approximation for the setting introduced above if the benefit associated to the hiring of \( HP \) workers are small (in the limit null) and the period in which old individuals are forced to retire is negligible.
References


